EXACT GROUP DELAY SENSITIVITIES FOR MINIMIZATION OF DELAY AND DISTORTION IN HIGH-SPEED VLSI INTERCONNECTS

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Abstract- Efficient computation of exact group delay and its sensitivities w.r.t. design parameters in multiconductor transmission line networks is presented. This method is combined with minimax optimization to perform gradient based minimization of delay and distortion in high-speed VLSI interconnects.

I. INTRODUCTION

As the signal speed increases, the effects of VLSI interconnects such as delay, distortion and crosstalk become the dominant factor limiting the performance of the overall VLSI system. With subnanosecond rise times, the electrical length of the interconnections becomes a significant fraction of the signal wavelength. Consequently distributed and lossy transmission line models must be used. There is a thrust of research in the time-domain analysis of such interconnect effects, e.g., [1-6]. Design optimization of interconnects is addressed very recently[7], where transient responses are improved by time domain optimization.

In this paper we present an alternative approach to minimization of transient responses such as delay, distortion and crosstalk by using frequency domain information such as group delay.

Group delay and its sensitivities have been an attractive vehicle for circuit design such as design of filter and IC digital cells[8-10]. However for lossy multiconductor transmission line networks, group delay sensitivity is much more involved and has not been previously presented.

The purpose of this paper is two fold. Firstly, a technique for efficient computation of exact group delay and its sensitivities is derived for multiconductor transmission line networks. Secondly, the group delay information is combined with gradient based minimax optimization to minimize delay and distortion in VLSI interconnects.

II. NETWORK EQUATIONS WITH MULTICONDUCTOR TRANSMISSION LINES

The admittance matrix of a multiconductor transmission line required in a modified nodal equation of the overall circuit has been described in detail in[1]. Suppose the network π consists of lumped elements and N_s multiconductor transmission lines. The modified nodal equations for the overall network π are

$$\mathbf{C}_\pi \frac{d\mathbf{v}_\pi(t)}{dt} + \mathbf{G}_\pi \mathbf{v}_\pi(t) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{i}_k(t) = \mathbf{e}_\pi(t), \quad (1)$$

where $\mathbf{C}_\pi$ and $\mathbf{G}_\pi$ are N_π by N_π matrices determined by lumped elements in the network. $\mathbf{v}_\pi(t)$ is the vector of node voltage waveforms appended by independent voltage source currents and inductor current waveforms. $\mathbf{D}_k$ is an incidence matrix containing 1's and 0's which maps $\mathbf{i}_k(t)$, the terminal current waveform of the kth distributed transmission line, into the N_π-node space of network π. $\mathbf{e}_\pi(t)$ is the vector of source waveforms.

The s-domain equation is obtained by taking Laplace transform of (1)

$$\mathbf{Y}_\pi \mathbf{V}_\pi(s) = \mathbf{E}_\pi(s) + \mathbf{C}_\pi \mathbf{v}_\pi(0), \quad (2)$$

where

$$\mathbf{Y}_\pi = \mathbf{G}_\pi + s\mathbf{C}_\pi + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{A}_k \mathbf{D}_k^T, \quad (3)$$

and $\mathbf{A}_k$ is the nodal admittance matrix of the kth distributed transmission line.

The lossy multiconductor transmission line is assumed to be uniform along its length with an arbitrary cross section. The cross section of an N_k -conductor transmission line can be described by per unit length impedance and admittance matrices $\mathbf{Z}_L$ and $\mathbf{Y}_L$, respectively. $\mathbf{Z}_L$ and $\mathbf{Y}_L$ can be computed from physical/geometrical parameters of the transmission line through quasi-static analysis [11] or empirical formulas [12]. Let γ_i^2 be an eigenvalue of the matrix $\mathbf{Z}_L\mathbf{Y}_L$ with an associated eigenvector $\mathbf{x}_i$. The nodal admittance matrix for the multiconductor transmission line is[1]

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} \\ \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} \end{bmatrix}, \quad (4)$$

where

$$\mathbf{E}_1 = \text{diag.}\{\frac{1+e^{-2\gamma_i l}}{1-e^{-2\gamma_i l}},\ i = 1,2,\ldots,N_k\}, \quad (5)$$

$$\mathbf{E}_2 = \text{diag.}\{\frac{2}{e^{-\gamma_i l}-e^{\gamma_i l}},\ i = 1,2,\ldots,N_k\} \quad (6)$$

and l is the length of the transmission line. $\mathbf{S}_v$ is a matrix containing all eigenvectors $\mathbf{x}_i$, $i = 1,2,\ldots,N_k$. $\mathbf{\Gamma}$ is a diagonal matrix with $\Gamma_{i,i} = \gamma_i$, and $\mathbf{S}_i = \mathbf{Z}_L^{-1}\mathbf{S}_v\mathbf{\Gamma}$.

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III. COMPUTATION OF EXACT GROUP DELAY AND ITS SENSITIVITIES

A. Computation of Group Delay

Let

$$V_{out}(s) = \mathbf{u}^T \mathbf{V}_\pi(s) \tag{7}$$

be the response of interest in the s-domain, where $\mathbf{u}$ is a constant N_π -vector. Group delay can be defined as[8,9]

$$T_G = -Real\{\frac{1}{V_{out}}\frac{\partial V_{out}}{\partial s}\}. \tag{8}$$

To compute $\frac{\partial V_{out}}{\partial s}$, we use the adjoint sensitivity approach [8,9],

$$\begin{aligned}\frac{\partial V_{out}}{\partial s} &= (\mathbf{V}_\pi^a)^T[\frac{\partial \mathbf{Y}_\pi}{\partial s}\mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s}] \\ &= (\mathbf{V}_\pi^a)^T[(\mathbf{C}_\pi + \sum_{k=1}^{N_s}\mathbf{D}_k\frac{\partial \mathbf{A}_k}{\partial s}\mathbf{D}_k^T)\mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s}],\end{aligned} \tag{9}$$

where $\mathbf{V}_\pi^a$ is solved from adjoint equation

$$\mathbf{Y}_\pi^T\mathbf{V}_\pi^a = -\mathbf{u}. \tag{10}$$

To compute $\frac{\partial \mathbf{A}_k}{\partial s}$, we first compute the sensitivities of eigenvalues γ_i^2 and eigenvectors $\mathbf{x}_i$ w.r.t. frequency s by solving the linear equation

$$\mathbf{B}\begin{bmatrix}\frac{\partial \mathbf{x}_i}{\partial s} \\ \frac{\partial \gamma_i^2}{\partial s}\end{bmatrix} = \begin{bmatrix}\frac{\partial(\mathbf{Z}_L\mathbf{Y}_L)}{\partial s}\mathbf{x}_i \\ 0\end{bmatrix}, \tag{11}$$

where

$$\mathbf{B} = \begin{bmatrix}\gamma_i^2\mathbf{U} - \mathbf{Z}_L\mathbf{Y}_L & \mathbf{x}_i \\ \mathbf{x}_i^T & 0\end{bmatrix}. \tag{12}$$

The solution of (11) is used to obtain $\frac{\partial \mathbf{S}_v}{\partial s}$ and $\frac{\partial \mathbf{S}_i}{\partial s}$. The sensitivity $\frac{\partial \mathbf{A}_k}{\partial s}$ is then computed from:

$$\begin{aligned}\frac{\partial \mathbf{A}_k}{\partial s}\begin{bmatrix}\mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v\end{bmatrix} &= \begin{bmatrix}\frac{\partial \mathbf{S}_i}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial s}\end{bmatrix}\begin{bmatrix}\mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1\end{bmatrix} \\ &+ \begin{bmatrix}\mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i\end{bmatrix}\begin{bmatrix}\frac{\partial \mathbf{E}_1}{\partial s} & \frac{\partial \mathbf{E}_2}{\partial s} \\ \frac{\partial \mathbf{E}_2}{\partial s} & \frac{\partial \mathbf{E}_1}{\partial s}\end{bmatrix} - \mathbf{A}_k\begin{bmatrix}\frac{\partial \mathbf{S}_v}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial s}\end{bmatrix}.\end{aligned} \tag{13}$$

B. Computation of Group Delay Sensitivity

Let ϕ be a design variable. In order to obtain the sensitivity for group delay, we differentiate (8)

$$\frac{\partial T_G}{\partial \phi} = Real\{\frac{1}{V_{out}^2}\frac{\partial V_{out}}{\partial \phi}\frac{\partial V_{out}}{\partial s} - \frac{1}{V_{out}}\frac{\partial^2 V_{out}}{\partial s \partial \phi}\}. \tag{14}$$

Since $\frac{\partial V_{out}}{\partial \phi}$ can be calculated according to [2], here the problem is to find $\frac{\partial^2 V_{out}}{\partial s \partial \phi}$. Differentiating (9) w.r.t. ϕ yields

$$\begin{aligned}\frac{\partial^2 V_{out}}{\partial s \partial \phi} &= \frac{\partial(\mathbf{V}_\pi^a)^T}{\partial \phi}[\frac{\partial \mathbf{Y}_\pi}{\partial s}\mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s}] \\ &+(\mathbf{V}_\pi^a)^T[\frac{\partial^2 \mathbf{Y}_\pi}{\partial s \partial \phi}\mathbf{V}_\pi + \frac{\partial \mathbf{Y}_\pi}{\partial s}\frac{\partial \mathbf{V}_\pi}{\partial \phi}],\end{aligned} \tag{15}$$

where sensitivity of adjoint response is solved from

$$\mathbf{Y}_\pi^T\frac{\partial \mathbf{V}_\pi^a}{\partial \phi} = -\frac{\partial(\mathbf{Y}_\pi^T)}{\partial \phi}\mathbf{V}_\pi^a \tag{16}$$

and the second-order sensitivity of $\mathbf{Y}_\pi$ is

$$\frac{\partial^2 \mathbf{Y}_\pi}{\partial s \partial \phi} = \frac{\partial \mathbf{C}_\pi}{\partial \phi} + \sum_{k=1}^{N_s}\mathbf{D}_k\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}\mathbf{D}_k^T. \tag{17}$$

Now the problem is to compute the second derivative $\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}$, which requires the calculation of second derivatives of eigenvalues and eigenvectors. From (11), we have

$$\begin{aligned}\mathbf{B}\begin{bmatrix}\frac{\partial^2 \mathbf{x}_i}{\partial s \partial \phi} \\ \frac{\partial^2 \gamma_i^2}{\partial s \partial \phi}\end{bmatrix} &= \begin{bmatrix}\frac{\partial^2(\mathbf{Z}_L\mathbf{Y}_L)}{\partial s \partial \phi}\mathbf{x}_i + \frac{\partial(\mathbf{Z}_L\mathbf{Y}_L)}{\partial s}\frac{\partial \mathbf{x}_i}{\partial \phi} \\ 0\end{bmatrix} \\ &-\frac{\partial \mathbf{B}}{\partial \phi}\begin{bmatrix}\frac{\partial \mathbf{x}_i}{\partial s} \\ \frac{\partial \gamma_i^2}{\partial s}\end{bmatrix},\end{aligned} \tag{18}$$

where the derivative $\frac{\partial \mathbf{B}}{\partial \phi}$ contains first order sensitivities $\frac{\partial \gamma_i^2}{\partial \phi}$, $\frac{\partial \mathbf{x}_i}{\partial \phi}$ and $\frac{\partial(\mathbf{Z}_L\mathbf{Y}_L)}{\partial \phi}$ which have already been solved in calculating $\frac{\partial V_{out}}{\partial \phi}$. The solution of linear equation (18) needs only forward/backward substitutions since the LU factors of $\mathbf{B}$ is available from solving (11).

Once we have $\frac{\partial^2 \mathbf{x}_i}{\partial s \partial \phi}$ and $\frac{\partial^2 \gamma_i^2}{\partial s \partial \phi}$, we can easily obtain $\frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi}$ and $\frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi}$. Finally from (13), $\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}$ can be solved from

$$\begin{aligned}&\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}\begin{bmatrix}\mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v\end{bmatrix} = \begin{bmatrix}\frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi} & 0 \\ 0 & \frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi}\end{bmatrix}\begin{bmatrix}\mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1\end{bmatrix} + \\ &\begin{bmatrix}\frac{\partial \mathbf{S}_i}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial s}\end{bmatrix}\begin{bmatrix}\frac{\partial \mathbf{E}_1}{\partial \phi} & \frac{\partial \mathbf{E}_2}{\partial \phi} \\ \frac{\partial \mathbf{E}_2}{\partial \phi} & \frac{\partial \mathbf{E}_1}{\partial \phi}\end{bmatrix} + \begin{bmatrix}\frac{\partial \mathbf{S}_i}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial \phi}\end{bmatrix}\begin{bmatrix}\frac{\partial \mathbf{E}_1}{\partial s} & \frac{\partial \mathbf{E}_2}{\partial s} \\ \frac{\partial \mathbf{E}_2}{\partial s} & \frac{\partial \mathbf{E}_1}{\partial s}\end{bmatrix} \\ &+\begin{bmatrix}\mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i\end{bmatrix}\begin{bmatrix}\frac{\partial^2 \mathbf{E}_1}{\partial s \partial \phi} & \frac{\partial^2 \mathbf{E}_2}{\partial s \partial \phi} \\ \frac{\partial^2 \mathbf{E}_2}{\partial s \partial \phi} & \frac{\partial^2 \mathbf{E}_1}{\partial s \partial \phi}\end{bmatrix} - \frac{\partial \mathbf{A}_k}{\partial \phi}\begin{bmatrix}\frac{\partial \mathbf{S}_v}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial s}\end{bmatrix} \\ &-\mathbf{A}_k\begin{bmatrix}\frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi} & 0 \\ 0 & \frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi}\end{bmatrix} - \frac{\partial \mathbf{A}_k}{\partial s}\begin{bmatrix}\frac{\partial \mathbf{S}_v}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial \phi}\end{bmatrix}.\end{aligned} \tag{19}$$

The formulas for group delay and its sensitivity analysis are implemented in a program for the design of distributed networks with lossy coupled transmission lines. Sensitivities computed from the new formulas were compared with that from perturbation for the 3 transmission line example of Fig. 1. Excellent agreement was observed. Computation speed by the new formulas is faster than by perturbation.

IV. FORMULATION OF INTERCONNECT OPTIMIZATION

It is well known that the group delay contains information of signal propagation delay. For RC networks Vlach *et al.* recently verified group delay at frequency zero as being exactly equal to Elmore delay and observed an empirical relation between group delay and signal delay of responses corresponding to a step excitation[10]. We take advantage of group delay information for reduction of signal propagation delay in a distributed network environment. The actual optimization is performed in the frequency domain.

Let $\mathbf{\Phi}$ denote all design variables including physical/geometrical parameters of the transmission lines and parameters in termination/matching networks. Let $S_{Gu}(s_k)$ and

$S_{Gl}(s_k)$ be upper and lower specifications on group delay T_G, respectively. Let W_G be a positive weighting factor. The minimization of signal delay can be achieved by minimizing

$$W_G(T_G(\mathbf{\Phi}, s_k) - S_{G,u}(s_k)) \tag{20}$$

over a range of frequencies, $s_k = j\omega_k, k = 1, 2, \ldots, K$. By imposing a lower specification on group delay, we have error function

$$-W_G(T_G(\mathbf{\Phi}, s_k) - S_{G,l}(s_k)) \tag{21}$$

A typical lower specification is $S_{G,l}(s_k) = 0$. Simultaneous minimization of (20) and (21) over a frequency range reduces the group delay and improves the flatness of group delay. A flat group delay contributes to the reduction of signal distortion.

Let $F(\mathbf{\Phi}, s_k)$ be the transfer function of the network. Let $S_{F,l}(s_k)$ be the lower specification on the magnitude of $F(\mathbf{\Phi}, s_k)$ and W_F be a positive weighting factor. Minimizing the following error functions

$$-W_F[|F(\mathbf{\Phi}, s_k)| - S_{F,l}(s_k)] \tag{22}$$

increases the response signal level.

Finally, reduction of crosstalk is realized by minimizing the magnitude of crosstalk spectrum $V(\mathbf{\Phi}, s_k)$ using

$$W_C(|V(\mathbf{\Phi}, s_k)| - S_C(s_k)), \tag{23}$$

where $S_C(s_k)$ is the specification on the magnitude of spectrum $V(\mathbf{\Phi}, s_k)$.

Let $\mathbf{e}(\mathbf{\Phi})$ be a m-vector containing all necessary error functions in the form of (20)-(23). The optimization problem is to find $\mathbf{\Phi}$ such that

$$U(\mathbf{\Phi}) = \text{maximum}\{e_1(\mathbf{\Phi}), e_2(\mathbf{\Phi}), \ldots, e_m(\mathbf{\Phi})\} \tag{24}$$

is minimized subject to electrical and physical constraints $\mathbf{g}(\mathbf{\Phi}) \leq \mathbf{0}$ and $\mathbf{h}(\mathbf{\Phi}) = \mathbf{0}$. The constrains represent design rules. For example the total length of several interconnect lines must be constrained by the physical dimensions of the circuit board. The total separation between several coupled conductors must be limited by the geometrical space available to them. The minimax optimization of (24) is solved by a gradient-based two stage minimax algorithm[13]. The derivatives of $\mathbf{e}$ w.r.t. $\mathbf{\Phi}$ required by the optimizer is obtained by the new approach of sensitivity analysis described.

V. EXAMPLES

Example 1: 3 Transmission Line Network

The circuit of Fig. 1 is excited by a 6ns trapezoidal signal. The time responses before optimization are plotted in Fig. 2. The objective is to reduce signal delay of V_{out1} and V_{out2}, and the crosstalk voltages V_{cross1} and V_{cross2}, respectively. Design variables $\mathbf{\Phi}$ include capacitors C_2 and C_3, resistor R_9, lengths of the three transmission lines l_1, l_2 and l_3, distance between the 2 conductors d and the width of the conductors w. The initial variable values are

$$\begin{aligned} \mathbf{\Phi} &= [l_1 \ l_2 \ l_3 \ d \ w \ C_2 \ C_3 \ R_9]^T \\ &= [50 \ 40 \ 30 \ 2.49 \ 0.58 \ 2 \ 1 \ 50]^T, \end{aligned}$$

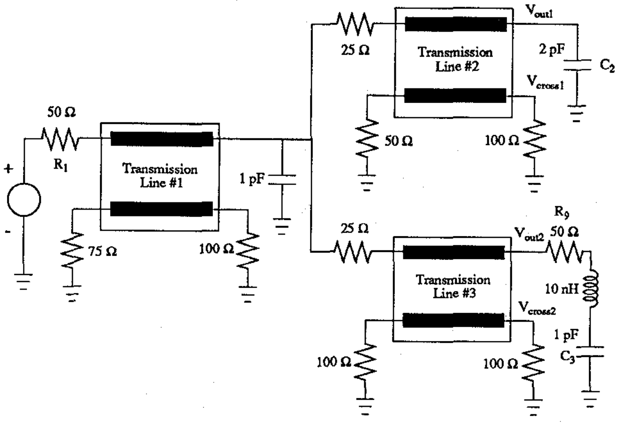

Fig. 1 Circuit schematic for the 3 transmission line network example.

where the unit for length, width and distance is mm. The units for capacitors and resisters are pF and Ω, respectively. The total length of the three transmission lines is fixed at 120mm and the width of each conductor plus the spacing between them is fixed at 3.07mm. The specifications on group delay and crosstalk spectrum are shown in Fig. 3. These specifications are imposed at 20 frequency points, $s_k = jk\omega_1$, $k = 1, 2, \ldots, 20$ and $\omega_1 = 0.0837758 \times 10^9$rad/s. After optimization, the objective function (24) was reduced from 1.0134 to -0.0386. The variables after optimization are

$$\mathbf{\Phi} = [98.38 \ 11.62 \ 10 \ 2.97 \ 0.1 \ 0.1 \ 8.884 \ 197.8]^T.$$

The group delay after optimization, as plotted in Fig. 3, is much lower and flatter than before optimization. Time responses after optimization are plotted in Fig. 2. The propagation delay times for V_{out1} and V_{out2} are both reduced from 1.6ns to 1.2ns. The magnitude of crosstalk signals V_{cross1} and V_{cross2} are both reduced by more than 55%.

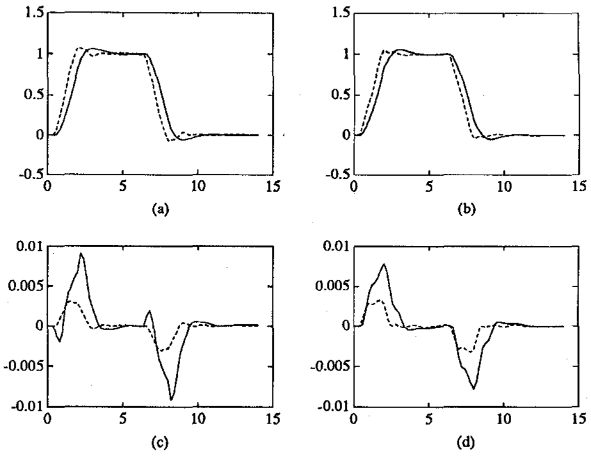

Fig. 2 The 3 transmission line example. Transient voltages(volts) vs. time(ns) before (solid line) and after (dashed line) optimization. (a). V_{out1}, (b). V_{out2}, (c). V_{cross1} and (d). V_{cross2}.

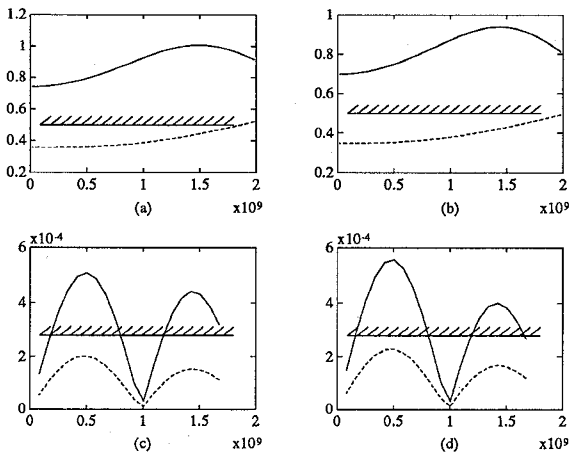

Fig. 3 The 3 transmission line example. Group delay and spectrum responses vs. angular frequency(rad/s) before (solid line) and after (dashed line) optimization. (a). T_G for V_{out1}, (b). T_G for V_{out2}, (c). spectrum for V_{cross1} and (d). spectrum for V_{cross2}.

Example 2: 25 Transmission Line Network

The frequency domain optimization approach is also applied to a 25 transmission line network which consists of 13 4-conductor transmission lines and 12 single-conductor transmission lines. There are 27 design variables including the lengths of the 4-conductor transmission lines, the distances between the conductors, 7 terminating resistances and 4 terminating capacitances as described in [7]. This circuit example was optimized using a time-domain optimization approach in [7]. Here we use frequency domain optimization with specifications on group delay and spectrum of crosstalks. The total number of error functions is 120 and the total number of linear constraints is 43. After 12 iterations of optimization, the objective function (24) was reduced from 22.908 to -1.177. At the frequency domain solution, the corresponding transient responses of the network are comparable to those from direct time domain optimization[7]. The propagation delays and the peaks of crosstalk voltages were reduced by more than 40% and 87%, respectively.

The CPU times for our approach and direct time domain approach[7] are 4519 and 15410 seconds(on SPARCstation 2), respectively. A CPU speedup factor of 3.4 is achieved by the new approach.

VI. CONCLUSION

An efficient method is presented for the computation of exact group delay and its sensitivities with respect to design parameters in multiconductor transmission line networks. By combining this method with minimax optimization, a frequency-domain approach is developed to indirectly minimize delay, distortion and crosstalk of transient responses in high-speed VLSI interconnects. It can be used as an efficient way in the design of interconnects in high-speed VLSI systems.

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